BUDDHASERIES

(Unit Wise Solved Questions & Answers)

Course–B.Tech(1st year)

College–Buddha Institute of Technology
(AKTUCODE-525)

Department:Applied Science and Humanities
Subject:EngineeringPhysics(BAS101)

Faculty Name:YAMAN KHAN YUSUF ZAI

(EM Theory)

Q. 1. Write maxwell's equation in integral and differential forms. Explain the physical significance of each equation.

Sol.

Maxwell's equations in Differential forms

1. $$\vec{\nabla}\cdot\vec{D} = \rho$$

2. $$\vec{\nabla}\cdot\vec{B} = 0$$

3. $$\vec{\nabla}\times\vec{E} = \frac{\partial\vec{B}}{\partial t}$$

4. $$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

Maxwell's equation in integralf orms

1. $$\oint_S \vec{D}\, d\vec{S} = \int_V \rho \, . dV$$

2. $$\oint \vec{B}\cdot d\vec{S} = 0$$

3. $$\oint_C \vec{E}\cdot d\vec{l} = \int \frac{\partial B}{\partial t}\cdot d\vec{S}$$

4. $$\oint_C \vec{H}\cdot d\vec{l} = \int\left[\vec{J} + \frac{\partial\vec{D}}{\partial t}\right]\cdot d\vec{S}$$

The physical significance of the field equations is readily obtained from their mathematical statement in the integral form are :

(i) The total electric displacement through the surface enclosing a volume is equal to the total charge within the volume.

(ii) The net magnetic flux emerging through any closed surface is zero.

(iii) The electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

(iv) The magnetic force around a closed path is equal to the conduction current plus the displacement through any closed surface.

Q.2. State and prove continuity equation.

Sol. This equation is mathematical expression for the conversation of charge. According to which the total charge contained in an isolated system remains conserved or constant.

Let us assume that the charge density ρ is a function of time. We know that rate of change of charge constituted the current. i.e.,

$$i = -\frac{dq}{dt} \quad \text{...(i)}$$

Current can also be written as

$$i = \int_S \vec{J}\, d\vec{S} \quad \text{...(ii)}$$

Also, charge q in terms of charge density ρ can be written as

$$q = \int_V \rho \cdot dV \quad \text{...(iii)}$$

Equating equation (i) and (ii) and using equation (iii) we have,

$$\int_S \vec{J} \cdot d\vec{S} = -\frac{dq}{dt} = -\frac{d}{dt}\int_V \rho dV \quad \text{...(iv)}$$

using gauss divergence theorem, we have

$$\int_V \nabla \cdot \vec{J}\, dV = \int_S \vec{J} \cdot d\vec{S}$$

equation (iv) becomes

$$\int_V \vec{\nabla} \cdot \vec{J}\; dV = -\int_V \frac{\partial \rho}{\partial t}\, dV$$

or,

$$\int_V \left(\nabla \cdot J + \frac{\partial \rho}{\partial t}\right) dV = 0$$

Above equation is true for any arbitrary limit volume. Hence

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Above equation is known as equation of continuity. In case of stationary current, density at any point within the region should remain constant,

$$\frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{J} = 0$$

which indicates that there is not net outward flux of current density $\vec{J}$.

Q.3. Derive Maxwell equation. Explain Physics significance of each equations.

Sol. 1. Maxwell's first equation or Gauss's law in Electrostaties, According to Gauss law, the electric flux ϕ passing through any closed hypothetical surface in an electric field is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface *i.e.*,

$$\phi = \oint_S \vec{E} . d\vec{S} = \frac{q}{\varepsilon_0}$$

where ε_0 is permittivity of free space.

Let the charge be distributed over a volume and Let ρ be the volume charge density. then,

$$\oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\oint \rho \cdot dV$$

Using Gauss divergence theorem,

$$\oint E.dS = \oint \text{div } \vec{E}\, dV$$

Using Gauss divergence theorem

$$\oint E.dS = \oint \text{div } \vec{E}\, dV$$

Hence
$$\oint \text{div } \vec{E}\, dV = \frac{1}{\varepsilon_0}\oint_V \rho\, dV$$

This is true for any arbitrary volume of shape dV So,

$$\text{div } \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\varepsilon_0}$$

if $\vec{D} = \varepsilon_0\,\vec{E}$ then

$$\vec{\nabla} . \vec{E} = \frac{\rho}{\varepsilon_0}$$

This is maxwell first equation

Maxwell Second Equation

The magnetic field passing through a closed surface placed in magnetic field is zero. Thus,

$$\oint \vec{B} \cdot d\vec{S} = 0$$

By using Gauss divergence theorem

$$\oint \vec{B} \cdot d\vec{S} = \oint (\nabla \cdot B)\, dV$$

$$0 = \oint \text{div } B\, dV$$

$$\text{div } \vec{B} = 0$$

Maxwell's Third Equation

The magnetic field flux linked with the loop is given by

$$\phi = \oint_S \vec{B} \cdot d\vec{S}$$

The e.m.f voltage is related to $\vec{E}$

$$\text{e.m.f} = \oint_l \vec{E} \cdot d\vec{l}$$

Now,
$$\int \vec{E} \cdot d\vec{l} = \frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{S}$$

By using stoke's theorem,

$$\oint \vec{E} \cdot d\vec{l} = \int_S (\nabla \times E)\, dS$$

$$\int (\vec{\nabla} \times \vec{E}) \cdot dS = -\frac{\partial}{\partial t}\int d\vec{S}$$

Thus
$$(\vec{\nabla} \times \vec{E}) = -\frac{\partial B}{\partial t}$$

Maxwell's Fourth Equation

We know that

$$\int \vec{H} \cdot d\vec{l} = I$$

$$\int \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} \qquad \text{...(1)}$$

Applying stokes theorem, we have

$$\int \vec{H} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{H}) \cdot d\vec{S} \qquad \text{...(2)}$$

equating equation (1) and equation (2)

$$(\vec{\nabla} \times \vec{H}) = \vec{J}$$

Ampere's law $(\vec{\nabla} \times \vec{H}) = \vec{J}$ does't hold good for time varying field. Maxwell's Modified the above equation by introducing the concept of "displacement current".

$$\vec{\nabla} \times \vec{H} = \vec{J}$$

taking divergent

$$\vec{\nabla}.(\vec{\nabla} \times \vec{H}) = (\vec{\nabla}.\vec{J})$$

divergence of curl of a vector is always zero

i.e.,
$$\nabla \cdot J = 0$$

But $\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t}$ this is not true for time varying field

Taking Gauss law,

$$\vec{\nabla}.\vec{D} = \rho$$

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla.\frac{\partial E}{\partial t} = \frac{\partial \rho}{\partial t}$$

$$\varepsilon \nabla.\frac{\partial E}{\partial t} = \frac{\partial \rho}{\partial t}$$

According to general equation of continuity,

$$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Thus,

$$\vec{\nabla}.\vec{J} + \varepsilon \vec{\nabla}.\frac{\partial E}{\partial t} = 0$$

$$\vec{\nabla}.\left[J + \frac{\partial D}{\partial t}\right] = 0$$

$\left[J + \frac{\partial D}{\partial t}\right]$ is the total current density and modified form of Maxwell's fourth equation,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

where J = Current density

$\frac{\partial \vec{D}}{\partial t}$ = Displacement Current

Hence

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Maxwell's fourth equation.

Q.4. Explain the concept of Displacement current and show how it led to the modification of the Ampere's law

Sol. See solution (3)

Q.5. Explain the concept of Displacement current. Why and how Maxwell modified Ampere's law.

Sol. See Solution (03).

Q.6. What was inconsistency in Ampere's law before Maxwell and how makes it consistent ? Also Explain the role of displacement current ?

Sol. See Solution (03)

Q.7. Derive Maxwell Equation. Explain Physical significance of each equation.

Sol. See Solution (03).

Q.8. Using Maxwell's relation

$$\vec{\nabla}.\vec{D} = \rho \text{ and } \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Prove that

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0 \textbf{ equation of continuity}$$

Sol.

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial D}{\partial t}$$

$$\vec{\nabla}.(\vec{\nabla}.\vec{H}) = \vec{\nabla}.\vec{J} + \nabla\cdot\left(\frac{\partial D}{\partial t}\right)$$

$$0 = (\vec{\nabla}.\vec{J}) + \nabla.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$(\vec{\nabla}.\vec{J}) + \left(\frac{\partial \rho}{\partial t}\right) = 0$$

Proved

Q.9. State and explain Poynting theorem for flow of energy in emw.

Sol. According to Poynting theorem, the rate at which energy is transmitted through unit area perpendicular to the direction of propogation of energy is given by $\vec{p} = \vec{E}\times\vec{H}$

where $\vec{p}$ is called as Poynting vector

Derivation : Maxwell's Equation

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad ...(1)$$

$$\vec{\nabla}\times\vec{E} = \frac{-\partial \vec{B}}{\partial t} \qquad ...(2)$$

Taking dot product,

$$\vec{E}.(\vec{\nabla}.\vec{H}) = (\vec{E}.\vec{J}) + \vec{E}.\frac{\partial \vec{D}}{\partial t}$$

$$E\cdot(\vec{\nabla}.\vec{H}) = \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon E^2\right] \qquad ...(3)$$

and

$$\vec{H}.(\vec{\nabla}.\vec{H}) = H\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon E^2\right] \qquad ...(4)$$

Substraction equ (4) from equ (3)

$$\vec{E}.(\vec{\nabla}\times\vec{H}) - \vec{H}.(\vec{\nabla}.\vec{E}) = (\vec{E}.\vec{J}) + \frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] \quad ...(5)$$

But $$\vec{\nabla}.(\vec{\nabla}\times\vec{H}) = \vec{H}(\vec{\nabla}\times\vec{E}) - \vec{E}.(\vec{\nabla}\times\vec{H})$$

Equation (5) becomes

$$-\vec{\nabla}.(\vec{E}\times\vec{H}) = \vec{E}.\vec{J} + \frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] \quad ...(6)$$

Let us know consider a volume V enclosed by a surface S. Integrating the above relative over the volume V, we have

$$\frac{\partial}{\partial t}\int_V\left[\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right]dV + \int_V(\vec{E}.\vec{J})\,dV = \int_V(\vec{E}\times\vec{H})\,dV$$

using Gauss div theorem

$$\int_V\vec{\nabla}.(\vec{E}.\vec{H})\,dV = \int(\vec{E}.\vec{H})\cdot d\vec{S}$$

$\because$ $$-\frac{\partial}{\partial}t\int_V\left[\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right]dV - \int_V(\vec{E}.\vec{J})\,dV = \oint(\vec{E}.\vec{H})\,d\vec{S}$$

Hence $$-\frac{\partial}{\partial t}\int_r\left[\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right]dV - \int_V(\vec{E}.\vec{J})\,dV = \oint(\vec{E}.\vec{H})\,d\vec{S}$$

Here, $$-\frac{\partial}{\partial t}\int_r\left[\frac{\mu H^2}{2} + \frac{\mu E^2}{2}\right]dV$$

represents the rate of decrease of energy stored in volume V due to Electric and Magnetic field.

$-\int_V(E\cdot V)\,dV$ represents the rate at which electromagnetic energy is lost through Joule heating.

Hence $\oint_S(\vec{E}\times\vec{H})\,d\vec{s}$ represent the rate of flow of which energy over the surface S enclosing the volume V. Therefore $(\vec{E}\times\vec{H})$ gives rate of flow of energy through unit Area enclosing the volume V. this is denoted by $\vec{P}$ called Poynting vector

$$\vec{P} = \vec{E}\times\vec{H}$$

The direction of $\vec{P}$ is perpendicular to both $\vec{E}$ and $\vec{H}$.

Q.10. If the earth services 2 Cal min^{-1} cm^{-2} solar energy, what are the amplitudes of electric and magnetic field of radiation.

Sol.

Poynting vector,

$$\vec{p} = \vec{E} \times \vec{H}$$

$$p = EH \sin 90^\circ$$

$$p = EH$$

$$\text{Solar Energy} = 2 \text{ cal min}^{-1}\text{cm}^{-2}$$

$$\text{Solar Energy} = \frac{2 \times 4.18 \times 10^4}{60} \text{ Joule m}^{-2} \text{ sec}^{-1}$$

$$EH = \frac{2 \times 4.14 \times 10^8}{60}$$

$$EH = 1400$$

Now,

$$\frac{E}{H} = 377\Omega \qquad ...(2)$$

From equation (1) and equation (2)

$$EH \times \frac{E}{H} = 1400 \times 377$$

$$E^2 = (1400 \times 377)$$

$$E = 726.5 \text{ volt/m}$$

and

$$H = \frac{E}{377} = 1.927$$

Amplitude of $\vec{E}$ and $\vec{H}$

$$E_0 = \sqrt{2}E$$

$$E_0 = 1024.3 \text{ Volt/m}$$

and

$$H_0 = \mathbf{2.717 \text{ Amp-turn/metre}}$$

Q.11. If the upper atmospheric layer of earth receives 1360 wm^2 energy from the sun what will be the peak value of electric and magnetic fields at the layer.

Sol. The energy flux is given by

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

or

$$S = \frac{EB}{\mu_0} = \frac{E_{rms}^2}{\mu_0 C}$$

$$E_{rms} = \frac{E_0}{\sqrt{2}} \qquad \therefore S = \frac{E_0^2}{2\mu_0 C}$$

or

$$E_0 = \sqrt{2\mu_0 CS}$$

$$E_0 = \sqrt{2 \times 4\pi \times 10^{-7} \times 3 \times 10^8 \times 1360}$$

$$E_0 = 1.012 \text{ KVm}^{-1}$$

Peak value of MF

$$B_0 = \frac{E_0}{C} = \frac{1012}{3 \times 10^8}$$

$$B_0 = 3.37 \times 10^6 \text{ wbm}^{-2}$$

Q.12. Write Maxwell equation in the shape and using these equation derive wave equation for both electric and magnetic field.

Sol. Maxwell equations

$$\vec{\nabla}.\vec{D} = 0 \quad ...(1)$$

$$\vec{\nabla}.\vec{B} = 0 \quad ...(2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad ...(3)$$

$$\vec{\nabla} \times \vec{H} = -\frac{\partial D}{\partial t} \quad ...(4)$$

For free space $\mu = \mu_0, \varepsilon = \varepsilon_0, \nabla = 0$ and $\rho = 0$

And $\vec{D} = \varepsilon_0 \vec{E}$ and $\vec{B} = \mu_0 \vec{H}$

taking Curl of equation (iii) both sides

$$(\vec{\nabla} \times \vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times B)$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} \frac{\partial H}{\partial t}$$

$$-\nabla^2 E = -\mu_0 \frac{\partial^2 H}{\partial t^2} = -\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad ...(5)$$

Similarly, $$\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} \quad ...(6)$$

Equation (5) and (6) represent wave equation in free space, Also general wave equation can be written as

$$\nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \quad ...(7)$$

Comparing either equation (6) or equation (5) from equation (7) we have,

$$\frac{1}{c^2} = \mu_0 \varepsilon_0$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Since

$$\varepsilon_0 = 2\pi \times 10^{-7} \text{ H/m}$$

$$\mu_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$C = \frac{1}{\sqrt{2\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$$C = 3 \times 10^8 \text{ m/sec}$$

This confirms that Em wave travel with the speed of light in free space.

Q.13. Write Maxwell's equation in integral and differential form and Explain their physical significance show that the velocity of Plane Em wave in free space is given by

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Sol. See Previous Question

Q.14. Calculate the Magnesium of Poynting at the surface of the sun. Given that power radiated by sun = 5.4×10^{28} watts and radius of sun is 7×10^8 m

Sol. Power radiated,

$$P = S \times 4\pi R^2$$

and

$$S = \frac{P}{4\pi R^2}$$

$$S = \frac{5.4 \times 10^{28}}{4 \times 3.14 \times (7 \times 10^8)^2}$$

$$S = 8.99 \times 10^9 \text{ watt/m}^2$$

Q.15. If the Average distance between the sun and earth is 1.5×10^{11} m and the power radiated by the sun is 5.4×10^{28} watt. Find the average solar energy incident on the earth.

Sol.

$$S_E \cdot 4\pi r^2 = P$$

$$S_E = \frac{5.4 \times 10^{28}}{4 \times 3.14 \times (1.5 \times 10^{11})^2}$$

$$S_E = 1.91 \times 10^5 \text{ watt/m}^2$$

$$S_E = \frac{1.91 \times 10^5}{4.2 \times 10^4} = 4.54 \text{ cal/cm}^2\text{-mm}$$

Q.16. Prove that electromagnetic wave transverse in nature

Sol. The Plane wave solution can be written as

$$E(r, t) = E_0 e^{i\vec{K}\cdot\vec{r} - i\omega t} \quad \text{...(1)}$$

$$B(r, t) = B_0 e^{(iK\cdot r - i\omega t)} \quad \text{...(2)}$$

Taking divergence of equ (1), we get

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot(\vec{E}_0 \ e^{i\,(K\cdot r - \omega t)})$$

$$\vec{\nabla}\cdot\vec{E} = iK\cdot E_0 e^{i\,(K\cdot r - \omega t)}$$

$$\vec{\nabla}\cdot\vec{E} = i(\vec{K}\cdot\vec{E})$$

for free space,

$$\vec{\nabla}\cdot\vec{E} = 0$$

i.e., $$i(\vec{K}\cdot\vec{E}) = 0 \quad \text{...(3)}$$

Similarly $$i(\vec{K}\cdot\vec{B}) = 0 \quad \text{...(4)}$$

Equation (3) and (4) indicates that, electric field and magnetic field are perpendicular to the direction of propagation vector $\vec{K}$. That is Em waves are Transverse in nature.

Q.17. Show that *E*, *H* and direction of Propagation from a set of orthogonal vectors.

Sol. Maxwell's Equation

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{...(1)}$$

and $$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \quad \text{...(2)}$$

The plane wave solution is written as,

$$E = E_0 e^{i\,(\vec{K}.\vec{r} - \omega t)} \quad \text{...(3)}$$

and $$B = B_0 e^{i\,(\vec{K}.\vec{r} - \omega t)} \quad \text{...(4)}$$

Taking curl of equation (3), we get

$$\vec{\nabla}\times\vec{E} = \vec{\nabla}\times E^{i\,(K.r - \omega t)}$$

$$\vec{\nabla}\times\vec{E} = i\,(\vec{K}\times\vec{E}) \qquad ...(5)$$

Similarly

$$\vec{\nabla}\times\vec{B} = i(\vec{K}\times\vec{B}) \qquad ...(6)$$

Now from equation (v) and (i), we have

$$\vec{K}\times\vec{E} = \omega B$$

Similarly $\quad \vec{K}\times\vec{B} = -\omega\mu_0\varepsilon_0\,\vec{E}$

That is electric and magnetic vector are normal.

Q.18. Define skin effect.

Sol. The depth of Penetration is defined as the distance in which the strength of electric field associated with the em wave reduce due to attenuation by factor $\left(\frac{1}{e}\right)$ from its initial value

$$\text{Skin depth} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\text{Skin depth} = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

Q.19. Find δ at a frequency of 3×10^6 Hz in aluminium where $\sigma = 35.0\times10^6$ S/m and $\mu_r = 1$

Sol.

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

$$\delta = \frac{1}{\sqrt{3.14\times3\times10^6\times4\times3.14\times10^{-7}\times35\times10^6}}$$

$$\delta = 0.04716\,\mu\text{m}$$

Q. 20. Write down Maxwell's equation in free space and show that $\vec{E}$, $\vec{H}$ and direction of propagation vector form a set of orthogonal vectors. *[AKTU 2017]*

[**Hint :** Solution has been already given in question 17.]

Q. 21. What is the equation of continuity ? Obtain the required expression for it. Also give its physical significance ? *[AKTU 2017]*

[**Hint :** Solution has been already given in question 2.]

Q. 22. State and explain paynting theorem for the flow of energy in emW. *[AKTU 2017]*

[**Hint :** Solution has been already given in question 19.]

Q. 23. For silver $\mu = \mu_0$ and $\sigma = 3\times10^7$ mhos/mg. Calculate the skin depth at 10^8 Hz frequency. Given, $\mu_0 = 4\pi\times10^{-7}$ N/A^2. *[AKTU 2017]*

Sol.

$$\delta = \frac{1}{\sqrt{\pi f u \sigma}}$$

$$\delta = \frac{1}{\sqrt{\pi \times 10^8 \times 3 \times 10^7 \times 4\pi \times 10^{-7}}} = 9.19\,\mu\text{M}$$

Q. 24. What is displacement current ? *[AKTU 2017]*

Sol. A changing electric field in air field capacitor is equivalent to a current which flow as long as the electric field is changing. This equivalent current is know as displacement current. Displacement current is given as,

$$i_d = \frac{\partial D}{\partial t}$$

$$i_d = \varepsilon \frac{\partial \phi}{\partial t}$$

Here, D is known as displacement current density.

❑

BUDDHA SERIES

(Unit Wise Solved Questions & Answers)

Course–B.Tech(1styear)

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Department:Applied Science and Humanities

Subject:EngineeringPhysics(BAS101)

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(EM Theory)

→ Short Questions

Q.1 what is Compton effect and Compton shift. (21-22)

Ans. ⇒ The Phenamena of Scattering with change in frequency is called the compton effect. Compton effect is the out come of collision between the high energy Photon and free electron. During Collision the Photon transfer some energy to free electron. The rest energy Photon scattered in othe direction.

The frequency of Scattered Photon is always less than the frequency of incident Photon. This difference of frequency (or wavelength) is known as Compton shift.

$$\lambda_s = \lambda_0 + \frac{h}{m_0 c}(1-\cos\theta)$$

Q.2. Can Compton effect be observed with visible light? (22-23)

Ans. The Max. Compton shift is when $\theta = 180°$

$$(\Delta\lambda)_{Max} = \frac{h}{m_0 c}(1 \mp \cos 180°) = \frac{2h}{m_0 c} = 0.048 \text{Å}$$

For visible light the average value is

$$\lambda_{mean} \approx 5000 \text{Å}$$

where as the Max. Compton shift is 0.048 Å which is very very less than the average value of visible light wavelength. Thus Comp ton shift can not be observed in visible light.

Q.3. Write down Plancks expression for spectral energy density in Black Body radiation. [AKTU (2022-23)]

Ans. According to the Planck Law, the energy density of radiation in the frequency range ν and $\nu + d\nu$ is

$$U_\nu d\nu = \frac{8\pi h \nu^3 d\nu}{c^3 \left(e^{h\nu/KT} - 1\right)}$$

Q.4. Determine the wavelength of Photon. [AKTU (2018-19)]

Ans. The de Broglie wavelength $\lambda = \frac{hc}{E}$

For Photon $E = h\nu$ then $\lambda = \frac{hc}{h\nu} = \frac{c}{\nu}$

Knowing frequency (ν), wavelength may be calculated.

Q.5. What is Wein's Displacement Law.

Ans. According to wein's displacement Law, the wavelength λ_m for maximum radiation intensity decreases as the temp increases.

$$\lambda_m \times T = \text{Constant}$$

⇒ Numericals :

Q.1. The wavelength of an X-ray Photon is doubled on being scattered through 90° with a Carbon block in Compton experiment. Find out the wavelength of the incident Photon.

[AKTU (2022-23)]

Ans

The Compton Shift

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Here $\frac{h}{m_0 c} = 0.024\,A^\circ$ and $\lambda' = 2\lambda$, $\theta = 90^\circ$

Thus

$$2\lambda - \lambda = 0.024(1 - \cos 90^\circ)A^\circ$$

$$\lambda = 0.024\,A^\circ$$

Ans

Q.2 An electron is trapped in one dimensional box of Length $1\,A^\circ$. Find the amount of energy that must be supplied to excite the electron from ground state to first excited state.

[AKTU (2022-23)]

Ans.

The Energy of Trapped electron in one dimensional box

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$E_n = 6.03 \times 10^{-18} n^2 \text{ Joule}$$

$$= \frac{6.03 \times 10^{-18}}{1.6 \times 10^{-19}} n^2 \text{ eV}$$

$$= 38 n^2 \text{ eV}$$

For ground state $n = 1$, $E_1 = 38\,eV$

For 1st excited state $n = 2$, $E_2 = 38 \times 4 = 152\,eV$

Difference $= E_2 - E_1 = 152 - 38 = 114\,eV.$

Q.3 Calculate the de Broglie wavelength of an α-Particle accelerated through a potential difference of 200 volts.

Ans The de Broglie wavelength of a charge particle is, $\lambda = \frac{h}{2mqV}$

the charge of α-particle, $q = 2e = 3.2\times10^{-19}C$

mass of α particle, $m = 4\times m_p = 4\times1.67\times10^{-27}$ Kg

accelerated by voltage $V = 200$ Volt.

thus

$$\lambda = \frac{6.63\times10^{-34}}{2\times4\times1.67\times10^{-27}\times3.2\times10^{-19}\times200} m$$

$$= 7.16\times10^{-13} m$$

$$= 0.00716 A^{o}$$ Ans

Q.4 Find de Broglie wavelength of a 15 KeV electron.

Ans K.E of electron. $\frac{1}{2}mv^2 = 15 keV = 15\times10^{3}\times1.6\times10^{-19}$ Joule

$$= 24.0\times10^{-16} \text{ Joule}$$

or $v^2 = \frac{2\times24\times10^{-16}}{9.1\times10^{-31}} = 5.3\times10^{15}$

or $v = 7.3\times10^{7} m/sec.$

The momentum $p = mv = 9.1\times10^{-31}\times7.3\times10^{7}$

$$p = 6.57\times10^{-23} \text{ Kg m/sec}$$

The de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{6.57\times10^{-23}} m$$

$$= 1\times10^{-11} m = 0.1 A^{o}$$

LongQuestion

WhatiswaveparticleDuality.DerivePlanckRadiationFormulaforBlackbodyradiation.

[AKTU2019,2022)

⇒ Wave Particle Duality – The Phenamena like Photoelectric effect, Compton effect and, emission and absorbtion of radiation established that the EM waves are Particle in nature. Whereas the optical phenamena like interference, diffraction, Polarisation of light shows that the EM waves are wave in nature. Thus the "Electromagnatic radiation serves as Particle as well as wave."
There are two modes of transfer of energy, one is Particle and the other is wave.

⇒ Assumption of Quantum Theory of Radiation

In 1901 Max Planck explain successfully the distribution of energy for entire wavelength (shorter and Longer). The proposed hypothesis is known as Quantum theory. The assumptions of Quantum theory are

(i) The cavity of black body contains linear oscillator of molecular dimension which can vibrate at all possible frequencies.

(ii) The linear oscillator can not emit energy in continuous manner, but in multiple of a small unit called quanta (Photon). The Energy is descreate and given by

$$E_n = nh\nu$$

where h = Planck Const $= 6.67 \times 10^{-34}$ J-sec.
ν = frequency
n = integer, $= 0, 1, 2, 3, ----$

(iii) The oscillator do not emit or absorb radiation energy continously but only in a certain multiplies of Packets of $h\nu$, while jumping from one ~~stage~~ state to another. The exchange of energy takes place in form $0, h\nu, 2h\nu, 3h\nu$ ------- $nh\nu$.

⇒ Planck's Radiation Formula :-

The average energy of a Planck oscillator is given by
$$\bar{E} = \frac{h\nu}{e^{h\nu/KT}-1} \qquad \text{——(1)}$$
(In Classical Theory $\bar{E} = KT$ from Law of equipartition)

The number of oscillator per unit volume lying in the frequency range ν and $\nu + d\nu$, as obtained by Rayleigh-Jeans is
$$\frac{8\pi\nu^2 d\nu}{c^3}$$
The energy density of radiation U_ν in frequency range ν and $\nu + d\nu$ is

$$U_\nu d\nu = \text{No. of oscillator per volume} \times \text{average energy}$$
$$= \frac{8\pi\nu^2 d\nu}{c^3}\left(\frac{h\nu}{e^{h\nu/KT}-1}\right)$$
$$\boxed{U_\nu d\nu = \frac{8\pi h\nu^3 d\nu}{c^3(e^{h\nu/KT}-1)}} \qquad \text{——(2)}$$

This is Planck formula of radiation in frequency form

Again since $\nu = \frac{c}{\lambda}$ and $d\nu = -\frac{c}{\lambda^2}d\lambda$

eq^n (2) becomes
$$U_\lambda d\lambda = -\frac{8\pi h \left(\frac{c}{\lambda}\right)^3\left(-\frac{c}{\lambda^2}\right)d\nu}{c^3\left(e^{hc/\lambda KT}-1\right)} \qquad (\because \lambda \propto \tfrac{1}{\nu})$$
$$\boxed{U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{(e^{hc/\lambda KT}-1)}} \qquad \text{——(3)}$$

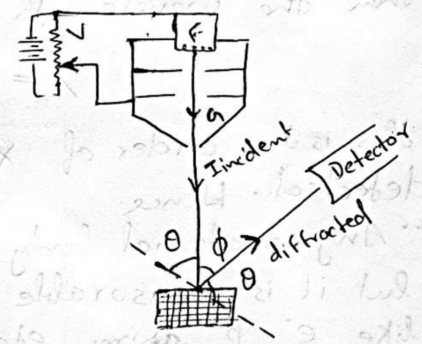
This is Planck formula of radiation in wavelength form.

DescribeDavissonGermerExperimenttodemonstratewaveparticleduality.

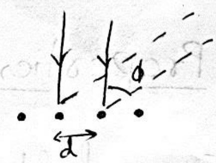
⇒ Davisson – Germer Experiment :

" The First experimental evidence that the stream of material particles show wave like properties was given by Davisson and Germer in 1927."

The electron beams are diffracted when they are scattered by the regular atomic array of crystals.



A beam of electon from hot cathod (Electron Gun) are accelerated through a potential difference V. These electron beam falls on a nickel crystal. The electrons are scattered in all direction by the atoms of crystal. These scattered electron are detected by detector.



By rotating the detector about an axis, the intensity of scattered electron is measured for different scattering angle.

The results : The graphs are plotted between scattering angle ϕ and intensity of scattered beam at different applying voltage

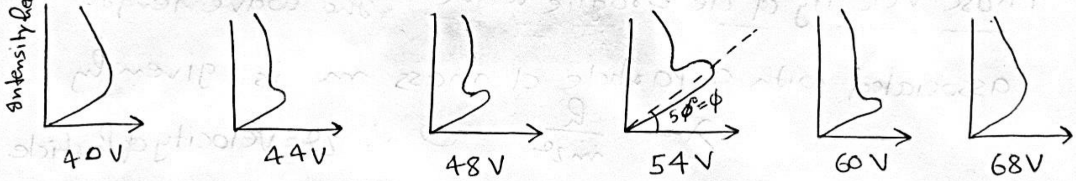
(i) Intensity of scattered electrons depends on angle of scattering ϕ

(ii) A Kink appears at $V = 44$ Volts

(iii) This Kink moves upwards as the voltage increases and becomes most prominent at $V = 54$ Volt. and at $\phi = 50°$

(iv) The size of Kink starts decreasing further on increasing Voltage and drops to zero at V = 68 Volt.

(V) The Kink at 54 volts gives the existence of electron waves



Calculation of wavelength:

The nickel Surface act as a grating diffraction element (with spacing d = 0.09 nm). According to the Bragg's Eqn

$$n\lambda = 2d\sin\theta$$

where θ = diffraction angle $= \dfrac{(180 - 50^\circ)}{2} = 65^\circ$

For 1st order (n = 1) $(\phi + 2\theta = 180)$

(and $\phi = 50^\circ$)

the wave length of Scattered electron wave

$$\lambda = \frac{2d\sin\theta}{n} = 2 \times 0.09\,\text{nm} \times \sin 65^\circ$$

$$\lambda = 1.65\,A^\circ \quad \text{—(1)}$$

According to De Broglie wave Eqn. the wave length of wave corresponding with electron is

$$\lambda = \frac{h}{\sqrt{2mE}} = \qquad \left(\because \lambda = \frac{12.27}{\sqrt{V}} A^\circ\right)$$

$$= \frac{6.6 \times 10^{-34}\,\text{J-s}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 54 \times 1.6 \times 10^{-19}}} \qquad E = eV$$

$$\lambda = 1.66\,A^\circ \quad \text{—(2)}$$

Eqn (1) and (2) agree. Thus the Davisson-Germer experiment varifies 'de-Broglie' hypothesis of wave nature of moving body.

Whatisgroupvelocity.Deriverelationbetweengroupvelocityandwavevelocity

'A wave Packet Consists of ~~waves~~ group of waves which is formed by superposition of ~~a~~ these waves Each wave in the Packet move with different velocity but the group of wave moves with different velocity which is Known as group velocity'.

"The velocity with which a wave Packet moves Forward in the medium is called group velocity"

Relation between Group Velocity and Phase (wave) Velocity

The Phase velocity of a single wave is given by

$$V_p = \frac{\omega}{K}$$

and the group velocity of a wave packet is given by

$$V_g = \frac{d\omega}{dK}$$

thus

$$V_g = \frac{d(V_p K)}{dK} = V_p + K\frac{dV_p}{dK}$$

$\because K = \frac{2\pi}{\lambda}$

$$V_g = V_p + \frac{2\pi}{\lambda} \cdot \frac{dV_p}{d\left(\frac{2\pi}{\lambda}\right)}$$

$$= V_p - \frac{2\pi}{\lambda} \cdot \frac{\lambda^2}{2\pi} \frac{dV_p}{d\lambda}$$

or $$\boxed{V_g = V_p - \lambda\frac{dV_p}{d\lambda}}$$

Cases:

(a) If $\frac{dv_p}{d\lambda} = 0$; when the Phase velocity does not depends on wavelength, then

$$\boxed{V_g = V_p}$$

This medium is called 'Non-dispersive'.

(b) If $\frac{dv_p}{d\lambda} =$ '+'ve then $V_g < v_p$ → normal dispersion

(c) If $\frac{dv}{d\lambda} =$ '−'ve then $V_g > v_p$ → Anomalous dispersion

DeriveSchrodinger time dependent andtime independent wave equation for matter wave.Givethephysicalinterpretationofwavefunction.[AKTU18-19,20-21,21-22,22-23]

Schrödinger Wave Equation: The Schrödinger's wave eq^n is the fundamental Equation of wave mechanics which is similar to the Newtons law of motion in Classical mechanics.

It is the differential Equation of the de Broglie wave associated with particles and describes the motion of Particles.

Time Dependent Eq^n :- For a free Particle the wave eq^n is

$$\psi = A e^{-\frac{i}{\hbar}(Et - px)}$$

where E = total Energy $= K.E. + P.E$

$$\therefore \quad E = \frac{p^2}{2m} + V$$

In the form of wave function, the total energy is written as

$$E\psi = \frac{p^2}{2m}\psi + V\psi$$

Substituting the operators $E = i\hbar\frac{\partial}{\partial t}$ and $p = -i\hbar\frac{\partial}{\partial x}$

we get $\cancel{i\hbar\frac{\partial}{\partial t}\psi = \left(\frac{-i\hbar}{2m}\right)}$

$$i\hbar\frac{\partial}{\partial t}\psi = \frac{\left(-i\hbar\frac{\partial}{\partial x}\right)^2}{2m}\psi + V\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi$$

If the Particle is moving in 3-dimensional space then

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

Schrödinger time independent wave Eqⁿ From time-dependent wave Eqⁿ (Imp)

The wave eqⁿ for a free Particle is stated as

$$\psi = A e^{-\frac{i}{\hbar}(Et - px)}$$

or $$\psi = A e^{-\frac{i}{\hbar}Et} \cdot e^{\frac{i}{\hbar}px}$$

If $$A e^{\frac{i}{\hbar}px} = \psi_0$$

then $$\psi = \psi_0 e^{-\frac{i}{\hbar}Et} \quad \text{--- (1)}$$

differentiating (1) w.r.t. 'x' twice

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi_0}{\partial x^2} \cdot e^{-\frac{i}{\hbar}Et} \quad \text{--- (2)}$$

differentiating (1) w.r.t. 't'

$$\frac{\partial \psi}{\partial t} = \psi_0 \left(-\frac{i}{\hbar}E\right) e^{-\frac{i}{\hbar}Et} \quad \text{--- (3)}$$

The Schrödinger time dependen eqⁿ is (for x direction)

$$\cancel{-\frac{\hbar^2}{2m}\nabla^2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar\frac{\partial \psi}{\partial t} \quad \text{--- (4)}$$

Substituting $\psi, \frac{\partial \psi}{\partial t}, \frac{\partial^2 \psi}{\partial x^2}$ from (1) (2) (3) in (4), we get

$$-\frac{\hbar^2}{2m} \cdot e^{-\frac{i}{\hbar}Et} \cdot \frac{\partial^2 \psi_0}{\partial x^2} + V \cdot \psi_0 \cdot e^{-\frac{i}{\hbar}Et} = i\hbar\,\psi_0\left(-\frac{i}{\hbar}E\right) e^{-\frac{i}{\hbar}Et}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_0}{\partial x^2} + V\psi_0 = E\psi_0$$

or $$\boxed{\frac{\partial^2 \psi_0}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi_0 = 0}$$

This is Schrodinger time-independent wave eqⁿ.

Wave Function and its Significance -1

"wave function is the quantity whose variation makes up the matter wave." It is similar to the height of water surface in water waves or similiar to the Pressure variation is sound wave or it is similiar to the Electric mag. field variation in light wave"
It is represented by ψ. The wave function is related to the Possibility of finding the Particle at particular Point (x, y, z) at the time 't'. It is a complex function and does not have a direct physical meaning.
(we can detect only that a particle is present or not. Present. we can not detect its exact Position and time)

$$\psi = A + iB$$

and $\psi^* = A - iB$ (conjugate of ψ)

and $\psi\psi^* = A^2 + B^2 = |\psi|^2$ $(\because i^2 = -1)$

"$|\psi|^2$ at a point is Proportional to the Probability of finding the particle at that time."

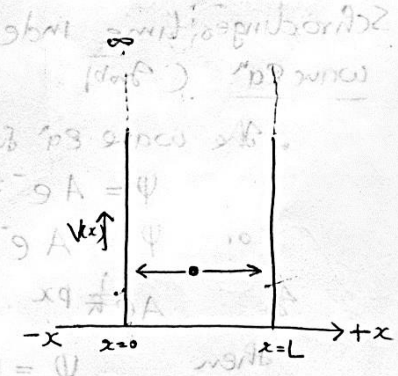
$$\boxed{\int_{-\infty}^{\infty} |\psi|^2 \, d\tau = 1} \longrightarrow \text{Normalised}$$

integral over the whole space is unity.

Solve Schrodinger Wave equation for a particle trapped in one dimensional potential box of length L. Calculate Eigen value and Eigen function.

Particle in a Box (Infinite Square Potential Well)
(An application of Schrödinger wave Eqⁿ)

Consider a particle trapped in a rectangular one dimension Potential Box. The particle moves along x-axis only and it bounces back and forth between the walls. The Particle does not lose any energy in bouncing. The P.E. V of the particle is zero inside the box, but it is infinite at the walls and outside the walls (the particle never comes outside)



The boundary conditions are

$V=0$ for $0<x<L$

$V=\infty$ for $x\leq 0$ and $x\geq L$ (L = length of Box)

The Schrödinger eqⁿ for the particle wave within the Box

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}E\psi=0$$

or

$$\frac{\partial^2\psi}{\partial x^2}+k^2\psi=0 \quad \text{—(1)} \qquad \text{where } k^2=\frac{2m}{\hbar^2}E$$

This is 2nd order differential eqⁿ and its solution given as

$$\psi=A\sin kx+B\cos kx \quad \text{—(2)}$$

Using Boundary Condition $\psi=0$ at $x=0$

$$0=A\sin 0+B \qquad \text{From (2)}$$

$$\Rightarrow B=0$$

Using Boundary Condition $\psi=0$ at $x=L$

$$\psi=A\sin kx \quad \text{—(3)}$$

$$0=A\sin kL$$

Since $A \neq 0$ then

$$\sin kL = 0 = \sin n\pi$$

or $k = \frac{n\pi}{L}$ $\therefore n = 1, 2, 3, \ldots$

Eq^n (3) becomes

$$\psi = A \sin \frac{n\pi}{L} x \quad \text{—— (4)}$$

The energy level of Particle is

$$E_n = \frac{k^2 \hbar^2}{2m}$$

$$= \left(\frac{n^2\pi^2}{L^2}\right) \cdot \left(\frac{h}{2\pi}\right)^2 \cdot \frac{1}{2m}$$

$$\boxed{E_n = \frac{n^2 h^2}{8 m L^2}} \quad \text{—— (5)}$$

$n = 1, 2, 3, \ldots$

" The Particle can have only certain descrete energy corresponding to $n = 1, 2, 3, \ldots$ which are known as Eigen values. The wave Function ψ corresponding to each eigen value are called eigen-function ψ_n"

Evaluation of Wave eigen-Function ψ

Applying Normalisation Condition

$$\int_{-\infty}^{\infty} |\psi_n(x)|^2 \, dx = 1$$

or $$\int_0^L |\psi_n|^2 dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

or $$\frac{A^2}{2} \int_0^L 2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

or $$\frac{A^2}{2} \left[\int_0^L \left\{1 - \cos\left(\frac{2n\pi x}{L}\right)\right\} dx \right] = 1$$

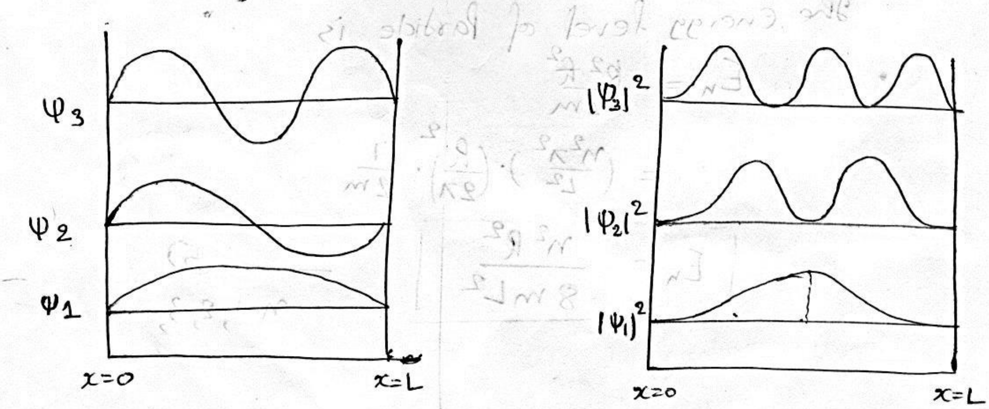
$$\frac{A^2}{2} \int_0^L dx - 0 = 1 \quad \therefore \int_0^L \cos \frac{2n\pi x}{L} dx = 0$$

or $$\frac{A^2}{2} L = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$$

thus the eigen function becomes

$$\boxed{\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x} \quad n = 1, 2, 3, \text{---}$$

ψ_n may be negative or Positive but $|\psi_n|^2$ is always Positive.



$n=1$ is ground state (minimum Energy Level)

Q

WhatisCompton Effect. DeriveCompton Shiftformula.

[AKTU18-19,21-22,22-23]

Compton effect

Compton effect shows the particle nature of radiation and also prove the quantization of energy.

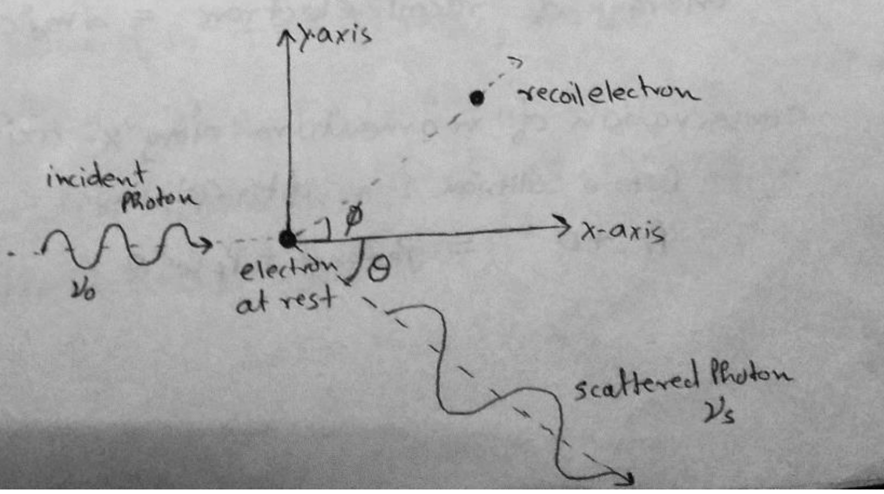
According to classical theory of radiation an electromagnetic wave of frequency ν_o incident on an atom, the electron of atom may oscillate with same frequency ν_o but in 1922 A.H. Compton found that when the radiation (Photon) of frequency ν_o incident on matter the electron of matter absorbs some energy and rest energy re radiate in the form of Scattered Photon with frequency ν_s $(\nu_s < \nu_o)$. The relation is given by

$$\boxed{\lambda_s = \lambda_o + \frac{h}{m_o c}(1 - \cos\theta)} \quad \text{or} \quad \boxed{\Delta\lambda = \lambda_s - \lambda_o = \frac{h}{m_o c}(1-\cos\theta)}$$

λ_s = wave length of Scattered Photon ; $\Delta\lambda$ = Compton Shift.
λ_o = wave length of incident Photon
m_o = mass of electron
θ = Scattering angle of Photon

$\frac{h}{m_o c}$ = Compton wavelength of electron $= 0.0024\,nm = 2.4\times10^{-2}\,A^\circ$

Proof: The Compton effect may Proved by Law of Conservation of momentum and energy.



a Photon of frequency ν_0 (energy $h\nu_0$ and momentum $P_i = \frac{h\nu_0}{c}$) incident on an rest electrom of mass m_0 after collision the electron get recoil and move away with energy ~~the~~ $\sqrt{m_0^2 c^4 + P_e^2 c^2}$ with angle ϕ. P_e is the momentum of recoil electron. The rest energy scattered in the form of scattered Photon of frequency ν_s (Energy $h\nu_s$ and momentum $P_f = \frac{h\nu_s}{c}$) with scattering angle θ.

Now Before Collision:

momentum of incident Photon $= P_i = \frac{h\nu_0}{c}$

momentum of ~~at~~ rest electron $= 0$

Energy of incident Photon $= h\nu_0$

Energy of rest electron $= m_0 c^2$

After Collision:

momentum of scattered Photon $= P_f = \frac{h\nu_s}{c}$

momentum of recoil electron $= P_e$

energy of scattered Photon $= h\nu_s$

energy of recoil electron $= \sqrt{m_0^2 c^4 + P_e^2 c^2}$

Conservation of momentum along x- axis

Before Collision $=$ after Collision

$$P_i + 0 = P_e \cos\phi + P_f \cos\theta \quad —①$$

Conservation of momentum along Y-axis

before Collision = after Collision

$$0 = P_e \sin\phi - P_f \sin\theta \qquad -(2)$$

from (1) and (2)

$$P_e \sin\phi = P_f \sin\theta \qquad -(3)$$

$$P_e \cos\phi = P_i - P_f \cos\theta \qquad -(4)$$

Squaring and adding (3) and (4)

$$P_e^2 = P_i^2 + P_f^2 - 2 P_i P_f \cos\theta \qquad -(5)$$

Applying Law of Conservation of energy

before Collision = after Collision

$$h\nu_0 + m_0c^2 = h\nu_s + \sqrt{m_0^2c^4 + P_e^2c^2}$$

or $$\left[(h\nu_0 - h\nu_s) + m_0c^2\right]^2 = m_0^2c^4 + P_e^2c^2$$

or
$$(h\nu_0 - h\nu_s)^2 + m_0^2c^4 + 2(h\nu_0 - h\nu_s)m_0c^2 = m_0^2c^4 + P_e^2c^2$$

or $$(h\nu_0 - h\nu_s)^2 + 2(h\nu_0 - h\nu_s)m_0c^2 = P_e^2c^2$$

or $$\left(\frac{h\nu_0}{c} - \frac{h\nu_s}{c}\right)^2 + 2m_0(h\nu_0 - h\nu_s) = P_e^2$$

or $$(P_i - P_f)^2 + 2m_0(h\nu_0 - h\nu_s) = P_e^2 \qquad -(6)$$

Substituting value of P_e^2 from (5) in (6), we get

$$(P_i - P_f)^2 + 2m_0(h\nu_0 - h\nu_s) = P_i^2 + P_f^2 - 2P_iP_f\cos\theta$$

or $$P_i^2 + P_f^2 - 2P_iP_f + 2m_0(h\nu_0 - h\nu_s) = P_i^2 + P_f^2 - 2P_iP_f\cos\theta$$

or $$P_iP_f(1 - \cos\theta) = m_0(h\nu_0 - h\nu_s)$$

or $$\frac{h\nu_0}{c} \cdot \frac{h\nu_s}{c}(1 - \cos\theta) = m_0 h(\nu_0 - \nu_s)$$

$$\frac{h}{m_0c^2}(1-\cos\theta) = \frac{(\nu_0-\nu_s)}{\nu_0\nu_s}$$

$$\frac{h}{m_0c^2}(1-\cos\theta) = \frac{1}{\nu_s} - \frac{1}{\nu_0}$$

or $$\frac{h}{m_0c}(1-\cos\theta) = \frac{c}{\nu_s} - \frac{c}{\nu_0}$$

or $$\frac{h}{m_0c}(1-\cos\theta) = \lambda_s - \lambda_0$$

or $$\boxed{\lambda_s = \lambda_0 + \frac{h}{m_0c}(1-\cos\theta)}$$

BUDDHA SERIES

(Unit Wise Solved Question & Answers)

Course–B.Tech(1styear)

College–Buddha Institute of Technology

(AKTUCODE-525)

Department:Applied Science and Humanities

Subject:Engineering Physics(BAS101)

Faculty Name:YAMAN KHAN YUSUF ZAI

Unit – 5 (Super Conductivity)

Superconductivity and Nanotechnology

- What ~~are~~ is superconductivity? Draw the curve of resistivity verses temperature for normal metal and Pure superconductors?

<u>Super Conductivity</u>:- "The phenamena of complete and sudden disappearance of electrical resistance in certain element or alloys below a certain temp (close to absolute zero) is known as superconductivity" The material which shows this property are called superconductor and the temp below which this property hold is called critical or transition temperature (T_c).

This phenamena was first observed by K. Onnes in 1911.

For example, Mercury suddenly becomes superconductor (resistivity 0.08 Ω) at 4K.

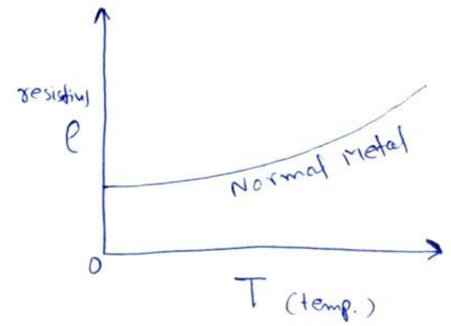
Liq. He becomes superconductor below 4.2K.

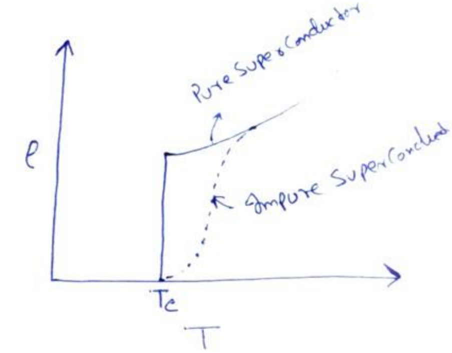
<u>Temperature Dependence of Resistivity in SuperConductor</u>

Generally resistivity of metal decreases on decreasing the temp. In ordinary metals the resistivity exist even at very low temp, but in superconductors the resistivity suddenly ~~becomes~~ vanishes at certain low temp. The critical temperature, below which a metal transform to superconductor is characteristic of material and is different for different metals

②

when the temperature of superconductor is increased the material transform into a normal metal
The Pure metal like Cu, Fe, Na do not show Superconductivity even at very low temp.





Q 2. What is Meissner Effect ? Prove that Meissner effect and the disappearance of resistivity in a superconductor are mutually consistent ?

Ans Meissner effect (or Effect of Magnetic field)

" The phenomena of Pushing the magnetic flux lines from the interior of superconductor when they cooled below Critical temperature is called Meissner effect."

Meissner found that normally when a specimen is placed in external magnetic field the magnetic Flux line passes through inside the specimen, but when it cooled below the critical temp. (T_c). The magnetic Flux pushed out of specimen and avoid to pass through it. The field lines do not pass inside the material.

Thus Below critical temp. the $\vec{B}=0$ (inside)

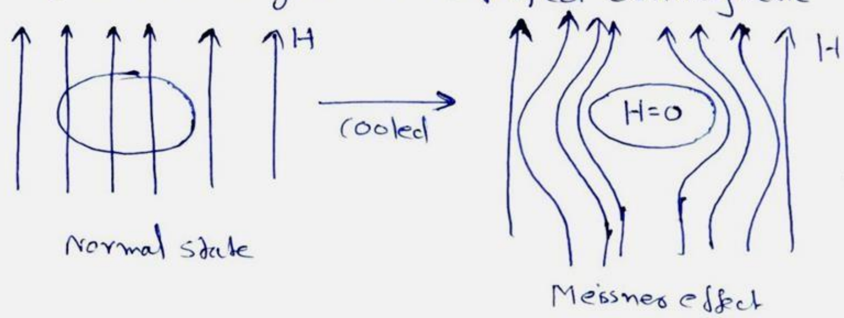
i.e. $B=\mu_0(H+M)=0 \Rightarrow M=-H$

and Magnetic Susceptibility $\chi=\frac{M}{H}=-1$

which is state of Perfect diamagnetism.

Therefore

"Superconducting state is perfect diamagnetic"



According to OHM's Law, the Electric field and resistivity (ρ) are related as

$$\vec{E}=\rho\vec{J}$$

for ρ to be zero (resistivity $=0$); the current density ($\vec{J}$) must be finite, and hence $\vec{E}=0$

From Maxwell's eq^n $\nabla \times E=-\frac{\partial B}{\partial t}$

For zero Resistivity ($\rho=0$) $E=0$

then

$$\frac{\partial B}{\partial t}=0 \Rightarrow B=\text{Const}$$

"Inside the superconductor the magnetic flux density is does not change even on cooling below T_c"

This is contradictionary to the Meissner effect ($B=0$)

Thus "Zero resistivity and Perfect diamagnetism are the two independent properties of superconductors"

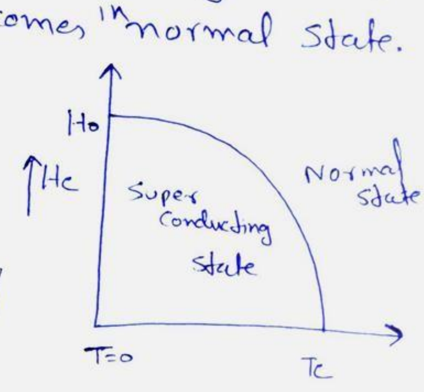
Q.3 What is Critical Field? Describe the effect of Magnetic Field on a superconductor.

Ans Critical Field :- "The minimum value of applied magnetic field when the material losses its superconducting state is called 'Critical Field' (H_c)".

. The necessary condition for superconductor is that the combination of temp. and field strength should be less than a critical value.

When the applied magnetic field exceeds the critical value H_c, the superconducting state is destroyed and material becomes in normal state.

H_c depends on the temperature. Value of H_c decreases as temp. increases from $T=0$ to $T=T_c$.



$$\boxed{H_c = H_0\left(1 - \frac{T^2}{T_c^2}\right)}$$

where H_c = Critical field strength at temp T.
H_0 = Max. field strength at absolute zero (0K)
T_c = Critical temp.

From the graph at any temp $T < T_c$, the material remains superconducting until a magnetic field is applied above H_c. When field strength $H > H_c$ the superconducting state is destroyed and material goes into normal state.

Numericals :

Q. 1 A superconducting tin has a critical temp 3·7K in zero mag. field and a critical field of 0·0306 Tesla at 0K. Find the critical field at 2K.

Ans. The critical field is given by $H_c = H_0 \left[1 - \frac{T^2}{T_c^2}\right]$

Here $H_0 = 0{\cdot}0306$ Tesla ; $T_c = 3{\cdot}7K$; $T = 2K$;

thus

$$H_c = 0{\cdot}0306 \left[1 - \left(\frac{2}{3{\cdot}7}\right)^2\right]$$

$$H_c = \underline{0{\cdot}0216 \text{ Tesla}}$$

Q.2 For a specimen of V_3Ga, the critical field are resp. 0·176 Tesla and 0·528 Tesla for 14K and 12K. Calculate the transition temp. and critical field at 0K and 4·2K.

Ans From the eq^n of critical field

$$H_c = H_0 \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Given at $T = 14K$ $H_0 = \cancel{0{\cdot}528}$ $0{\cdot}176$ Tesla

at $T = 12K$ $H_0 = 0{\cdot}528$ Tesla

thus

$$0{\cdot}176 = H_0 \left[1 - \left(\frac{14}{T_c}\right)^2\right] \quad —(1)$$

and

$$0{\cdot}528 = H_0 \left[1 - \left(\frac{12}{T_c}\right)^2\right] \quad —(2)$$

divide (2) by (1)

$$\frac{1 - \left(\frac{12}{T_c}\right)^2}{1 - \left(\frac{14}{T_c}\right)^2} = \frac{0{\cdot}528}{0{\cdot}176} \approx 3$$

Solving $T_c^2 = 222 \Rightarrow \underline{T_c = 14{\cdot}9K}$ Ans

from (1)

$$0{\cdot}176 = H_0 \left[1 - \left(\frac{14}{14{\cdot}9}\right)^2\right] \Rightarrow H_0 = \frac{0{\cdot}176}{0{\cdot}117} =$$

or $\underline{H_0 = 1{\cdot}5 \text{ Tesla}}$ Ans

What are Type I and Type II superconductors:

On the base of Magnetic Response, the superconductors are of two types

Type I (Ideal superconductor):

Those superconductor, which shows complete Meissner effect and Never allows the magnetic flux lines pass through their interior are called Type I or soft superconductors. They are perfectly diamagnetic in nature.

Type II (Hard Superconductor):

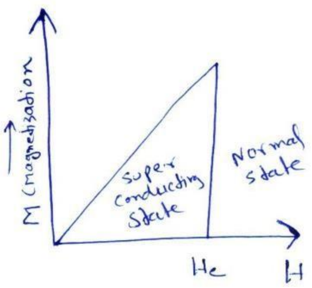
Those superconductor, which shows incomplete Meissner effect and partially allow to pass the magnetic flux lines ~~throu~~ interior through them are called Type II or hard superconductor.

The type II superconductor behaves differently in a magnetic field below T_c.

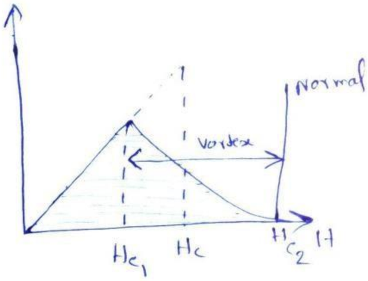
For a certain lower critical field H_{c_1} ($< H_c$) the superconductor is perfectly diamagnetic (No penetration of field through them)

On increasing magnetic field the field line start to pass through them and at a upper critical field H_{c_2} ($> H_c$) the field lines penetrate the material continuously. The region between H_{c_1} to H_{c_2} is known as 'Vortex State'. which a mixture state of superconductor and normal state.

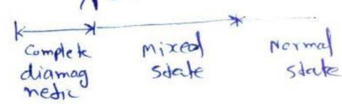
Examples of Type II are NiTi and Ni_3Sb.



Type I



Type II



A material can change from type I to type II superconductor by doping with some impurity. For example, when 20% of wt of indium is add to Lead which is type I superconductor.

Difference between Type I and Type II

	Type I	Type II
(i)	The superconductivity vanish suddenly at $H > H_c$	The superconductivity vanishes gradully upto an upper limit and then vanishes totally
(ii)	Coherent length is larger than the pentration depth	Coherent length is smaller than the pentration depth.
(iii)	The interface energy is positive	The interface energy is negative
(iv)	Low magnetic field produced ($\approx H_c = 0.1$ Tesla) (Not much used in practly)	High magnetic field produced (very much usefull in practically) like NMR, MRI, high speed trains, particle accelerator

Q.8 What are applications of Superconductors :

Ans Superconductors are widely used in many fields of Engineering includings radioelectronic, geophysics radio astronomy, medicine, high energy particle Physics. Superconductivity is a very low temp. Physics and known as 'Cryogenics'. Some of them are.

(i) Superconductors are used for Producing very ~~high~~ Strong magnetic field (≈ 50 Tesla)

(ii) High current density with zero resistance is used for strong electromagnet for MRI (magnetic resonance imaging) in medicine.

(iii) In Superconductor the heating loss is zero ($i^2R = 0$). therefore Power can be transmitted throught supercond-ding cable without loss

(iv) Type II superconductors are used for high speed trains (Mag lev)

(v) SQUID (Superconducting Quantum interface Device) are used in medicine which measures very weak fields generated by heart and brain.